



Rewarding Learning
ADVANCED
General Certificate of Education
2024

Further Mathematics

Assessment Unit A2 2
assessing
Applied Mathematics

[AFM21]



AFM21

WEDNESDAY 5 JUNE, AFTERNOON

TIME

2 hours 15 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the **Answer Booklet** provided.

You must answer the questions in the dedicated spaces provided in the Answer Booklet.

Do not write outside the boxed area on each page or on blank pages.

Complete in black ink only. **Do not write with a gel pen.**

Questions which require drawing or sketching should be completed using an HB pencil.

You must answer **all** questions from sections A and B **or** A and C **or** A and D **or** C and D.

You should spend equal time on each of the two sections.

Show clearly the full development of your answers. **Answers without working may not gain full credit.**

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 150.

The total mark for each section of this paper is 75.

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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SECTION A Mechanics 1

Answer all five questions in this section.

- 1 A designer creates a symmetrical sign to advertise bird houses, as shown in **Fig. 1** below.

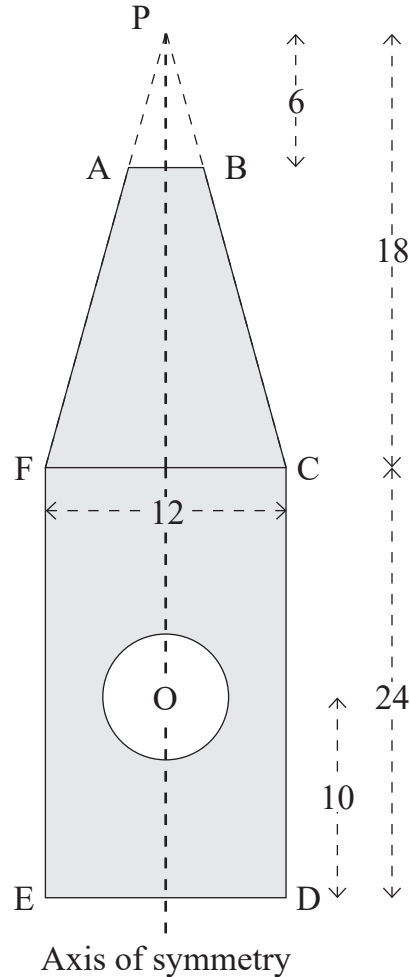


Fig. 1

The sign is modelled as a uniform lamina PCDEF from which an isosceles triangle PAB and a circle centred at O have been removed.

Triangle PAB Perpendicular height = 6 cm and AB = 4 cm

Triangle PFC Perpendicular height = 18 cm

Rectangle CDEF FC = 12 cm CD = 24 cm

The circle has radius 3 cm and its centre O is 10 cm above DE.

Let m be the mass per unit area in g cm^{-2}

Find the distance of the centre of mass of the sign from the base DE.

[9]

2 A framework of 5 light pin-jointed rods is shown in **Fig. 2** below.

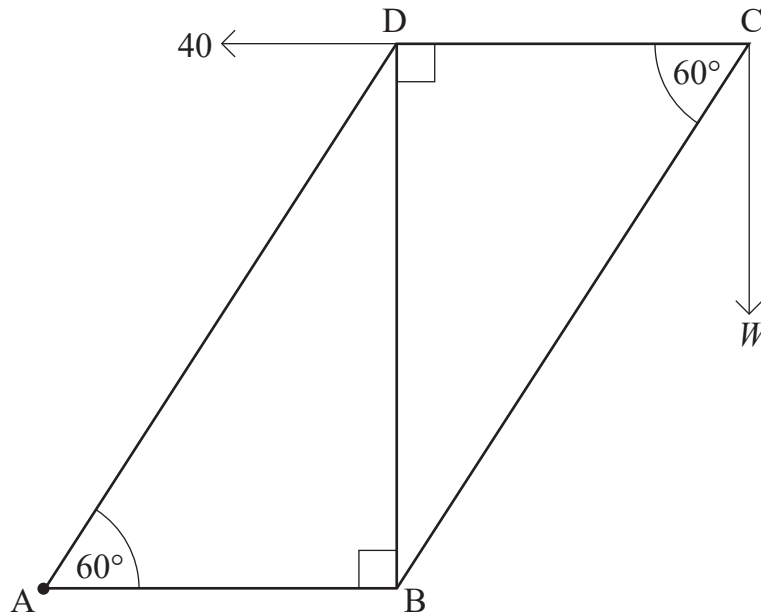


Fig. 2

The framework is hinged at A.

A horizontal force of 40 N acts at D.

A vertical force W N acts at C.

The framework is in equilibrium, in a vertical plane, with AB and DC horizontal.

$$\angle DAB = \angle DCB = 60^\circ$$

$$\angle ABD = \angle CDB = 90^\circ$$

$$AB = DC = l \text{ metres}$$

(i) Find the value of W . [4]

(ii) Find the magnitude and direction of the reaction acting on the framework at A. [7]

(iii) Find the forces in each of the rods AD and BD. [4]

(iv) State which of these two rods is in tension. [1]

3 A light elastic string has natural length l metres and modulus of elasticity $5mg$ newtons.

One end of the string is attached to a fixed point A.

A particle P of mass m kg is attached to the other end of the string.

P hangs in equilibrium vertically below A.

(i) Find the extension, e metres, of the string in this equilibrium position. [2]

P is pulled vertically downwards a further distance from the equilibrium position and then released from rest.

(ii) Show that at time t seconds after P is released, its equation of motion is given by

$$\frac{d^2x}{dt^2} = -\frac{5g}{l}x$$

where x metres is the displacement of P below the equilibrium position. [6]

(iii) State the maximum value of x for which P can continue to move with simple harmonic motion. [1]

(iv) Given that $l = 1.4$, and that $x = 0.2$ when $t = 0$, find the value of t when P first passes through the point where $x = 0.15$ [6]

4 [Take $g = 10 \text{ m s}^{-2}$ in this question.]

A particle of mass 0.5 kg is attached to one end of a light spring of natural length 0.8 m and modulus of elasticity 16 N .

The spring is hung from a fixed point O and the particle is suspended in a tank of liquid as shown in **Fig. 3** below.

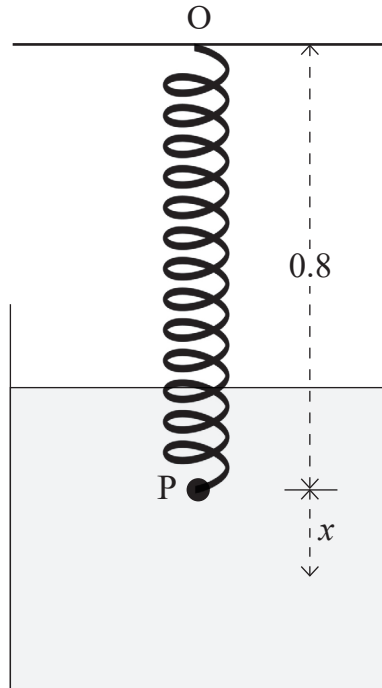


Fig. 3

The particle is released from rest at a point P which is 0.8 m below O .

The liquid provides a force of $-7v$ newtons where $v \text{ m s}^{-1}$ is the velocity of the particle at time t seconds.

(i) Show that the motion of the particle can be modelled by the equation

$$\frac{d^2x}{dt^2} + 14 \frac{dx}{dt} + 40x = 10$$

where x metres is the downwards displacement of the particle from P at time t . [4]

(ii) Determine whether this motion is under, over or critically damped. [4]

(iii) Solve the differential equation in (i) to find the displacement of the particle from P when $t = 1$ [9]

- 5 A lorry of mass M kg is travelling in a horizontal circle of radius r metres on a rough road banked at α to the horizontal as shown in Fig. 4 below.

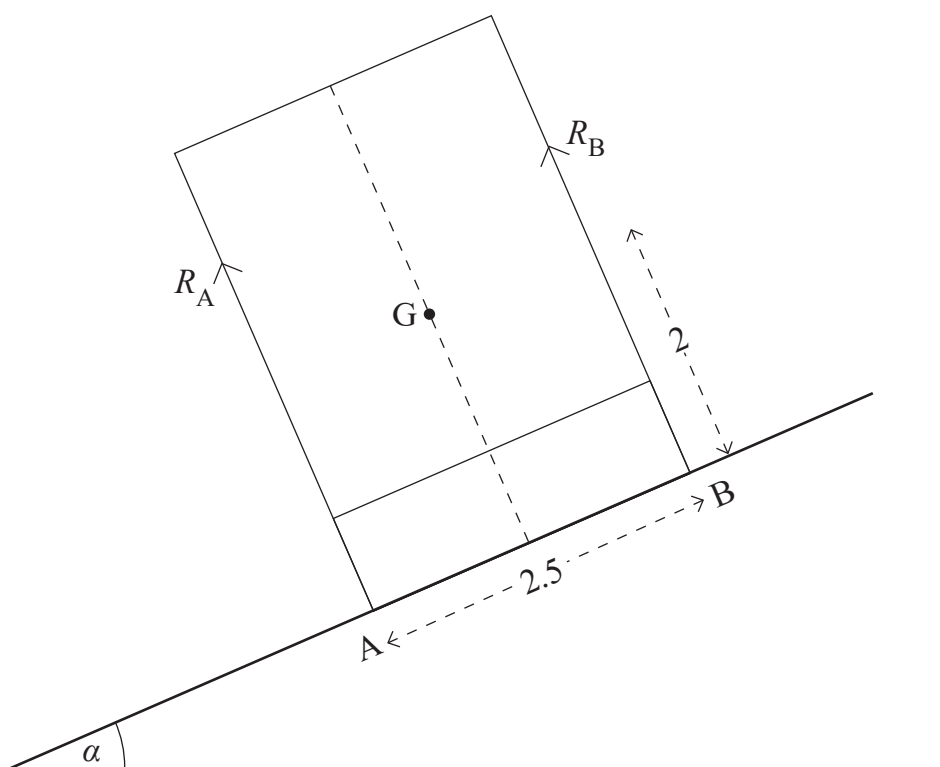


Fig. 4

The wheels, A and B, of the lorry are 2.5 m apart.

The centre of mass of the lorry, G, lies on its axis of symmetry at a height of 2 m above the road.

R_A newtons and R_B newtons are the normal reactions at A and B.

- (i) If the lorry is about to topple inwards when its speed is $v \text{ m s}^{-1}$, find an expression for v^2 in terms of r , g and α . [14]

If instead the lorry is about to slide down the road, then its speed $u \text{ m s}^{-1}$ is given by

$$u^2 = rg \left(\frac{\tan \alpha - \mu}{1 + \mu \tan \alpha} \right)$$

where μ is the coefficient of friction between the wheels and the road.

- (ii) If $\mu = 0.4$ and $\alpha = 25^\circ$, determine whether it is feasible for the lorry to be about to topple inwards, slide downwards or neither of these. [4]

SECTION B Mechanics 2

Answer all five questions in this section.

- 1 **Fig. 1** below shows a straight line through the points A, B, C and D.

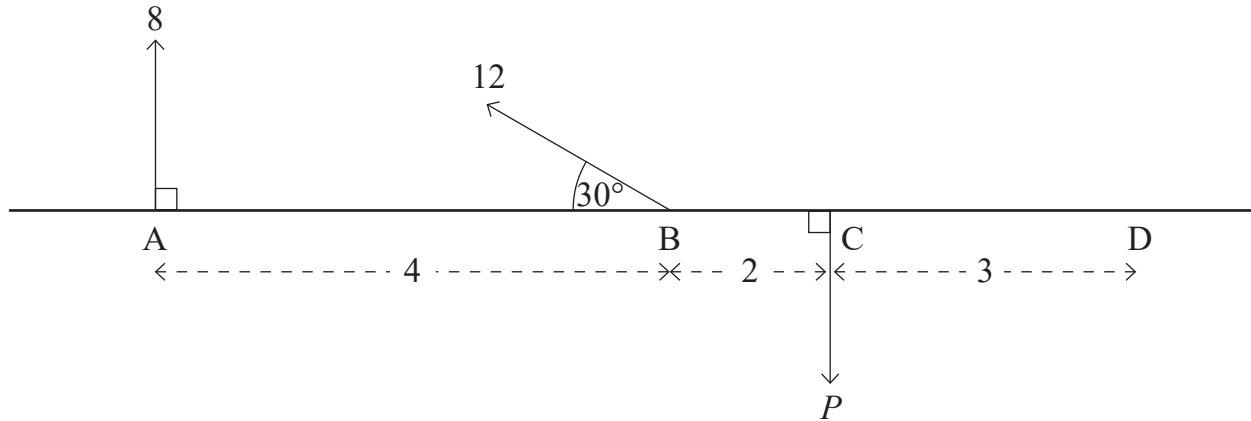


Fig. 1

$$AB = 4 \text{ m} \quad BC = 2 \text{ m} \quad CD = 3 \text{ m}$$

Forces of magnitude 8 N, 12 N and P newtons act at A, B and C respectively.

This system of forces is equivalent to a single force Q newtons acting at D.

- (i) Find the magnitude and direction of Q . [9]

- (ii) Find the value of P . [2]

- 2 At time $t = 0$ seconds a particle P is passing through a fixed point O with a velocity of $(7\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) \text{ m s}^{-1}$

The acceleration $\mathbf{a} \text{ m s}^{-2}$ of P at time t is given by

$$\mathbf{a} = \begin{cases} 4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k} & 0 \leq t \leq 2 \\ 7\mathbf{i} + \frac{16}{t^3}\mathbf{j} + 2t\mathbf{k} & 2 < t \leq 4 \\ 0 & t > 4 \end{cases}$$

- (i) Find the velocity of P at time t , when $2 < t \leq 4$ [8]

At time $t = 2$, P has a displacement of $(22\mathbf{i} + 10\mathbf{j} + 16\mathbf{k})$ metres from O.

- (ii) Find the displacement of P from O at time $t = 4$ [5]

- 3 Three identical spheres A, B and C, each of mass m kg, lie at rest on a smooth horizontal surface as shown in Fig. 2 below.

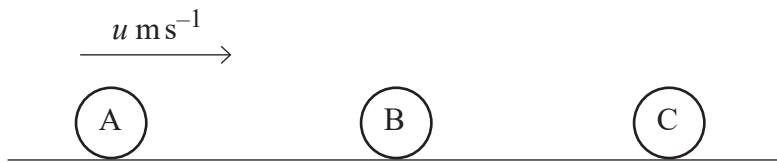


Fig. 2

A is projected towards B with a velocity $u \text{ ms}^{-1}$

A collides with B and then B collides with C.

The coefficient of restitution between each pair of spheres is $\frac{1}{2}$

- (i) Find the velocities of A, B and C after the collision of B with C. [11]
- (ii) Explain why A and B will collide for a second time. [1]
- (iii) Determine whether any further collisions will happen. [6]
- 4 For the first 6 seconds of its motion, a particle P of mass 4 kg moves in a straight line under the action of two horizontal forces, D newtons and R newtons.
- $D = 40t$ where t is the time in seconds.
- $R = -8(v + 1)$ where $v \text{ ms}^{-1}$ is the velocity of the particle at time t .
- D and R act in the same straight line as the motion of P.
- (i) Show that the equation of motion of P is
- $$\frac{dv}{dt} + 2v = 10t - 2 \quad [3]$$
- (ii) Given that $v = 0$ when $t = 0$, find the speed of P when $t = 6$ [13]

- 5 **Fig. 3** below shows a garden ornament which consists of a solid plastic cylinder from which a hemisphere has been removed. A solid glass hemisphere is fitted into this space.

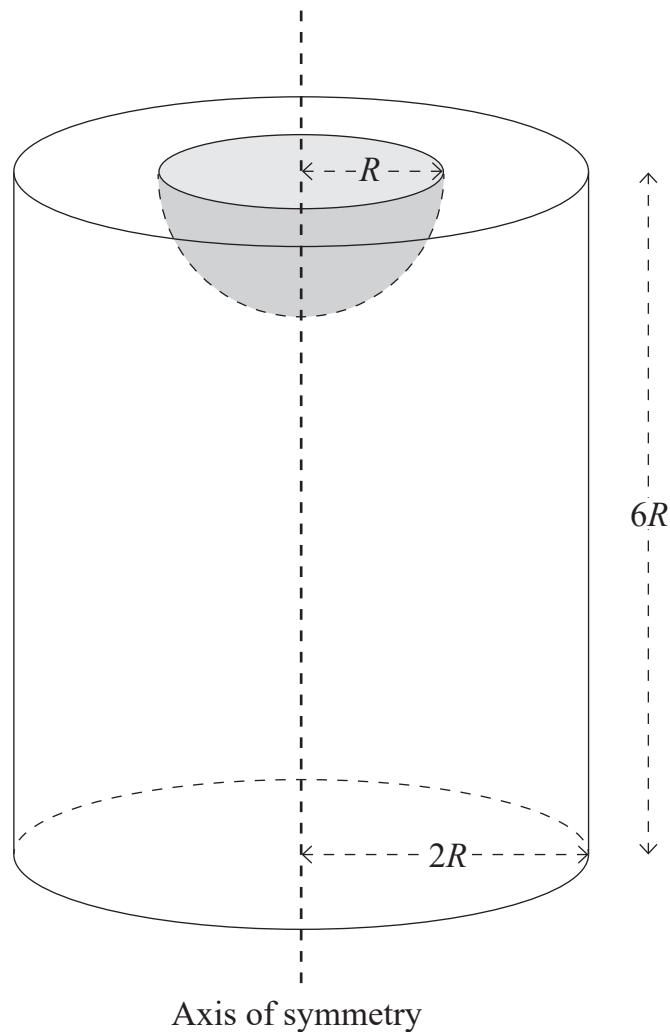


Fig. 3

The centre of the base of the hemisphere lies vertically above the centre of the base of the cylinder.

The plastic cylinder has radius $2R$ metres, height $6R$ metres and uniform density $\rho \text{ kg m}^{-3}$

The glass hemisphere has radius R metres and uniform density $3\rho \text{ kg m}^{-3}$

- (i) Find, in terms of R , the perpendicular height of the centre of mass of the ornament above the base of the cylinder. [10]

The ornament is placed on a slope which is inclined at 20° to the horizontal as shown in **Fig. 4** below.

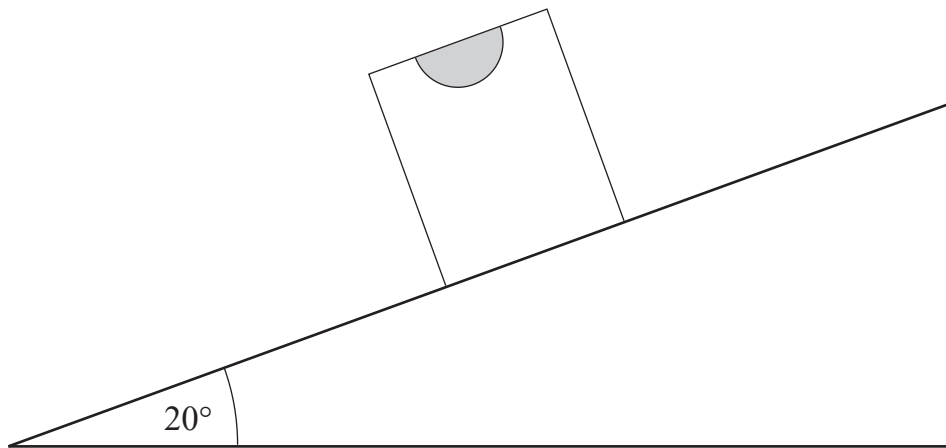


Fig. 4

The coefficient of friction between the ornament and the slope is 0.5

(ii) Determine whether the ornament slides, topples or does neither.

[7]

SECTION C Statistics

Answer all four questions in this section.

- 1 A zoologist is researching the differences between African and Asian elephants.

She identifies that the height of an African elephant, F feet, can be approximated by a Normal distribution with mean 14 and variance 2

The height of an Asian elephant, S feet, can be approximated by the distribution $N(10, 1)$.

- (i) By considering the variable $X = 2F - 3S$, or otherwise, find the probability that the height of a randomly selected Asian elephant will be less than two-thirds of that of a randomly selected African elephant. [9]

A random sample of 12 African elephants is selected.

Find the probability that:

- (ii) the total height of the 12 elephants exceeds 160 feet; [4]

- (iii) the mean height of the 12 elephants is less than 13.5 feet. [4]

- 2 A Chemistry teacher is investigating the uptake of his subject at Advanced Level in the current Year 13 cohort.

He found that, out of the 80 male students in the Year 13 cohort, 54 chose to study Chemistry. Out of the 120 female students in the Year 13 cohort, 64 chose to study Chemistry.

- (i) Carry out a suitable test, at a 10% level of significance, to determine whether or not the gender of the student and their likelihood to choose Advanced Level Chemistry are independent. [16]

- (ii) State, giving a reason, which gender the Chemistry teacher may wish to focus on when promoting the subject to next year's cohort. [2]

- 3 The description on a coffee machine claims that it dispenses cups of coffee in 160 ml portions.

The volumes of liquid, x ml, in a random sample of 8 cups of coffee were measured and produced the following summary statistics.

$$\sum x = 1282.4 \qquad \sum x^2 = 205586.08$$

The company that manufactures the coffee machine sends out an engineer to make refinements to its operation.

Following these refinements, the volumes, y ml, in a random sample of 10 cups of coffee were measured and produced the following summary statistics.

$$\sum y = 1600.8 \qquad \sum (y - \bar{y})^2 = 6.336$$

The company would like to check whether the average volume of liquid dispensed by the coffee machine has been affected by the refinements that were made.

- (i) Show that the pooled variance estimator is $S_p^2 = 1.481$ [7]
- (ii) Hence carry out a suitable t -test, at a 10% level of significance, to examine whether there is evidence that the refinements made to the machine have affected the average volume of liquid dispensed.
Assume the volumes are Normally distributed. [10]
- (iii) State the reason why it was appropriate to assume in part (ii) that the test statistic followed a t -distribution. [1]
- (iv) State the assumption that you have made when using S_p^2 in part (ii).
Give a reason why this assumption may not be valid in the context of this question. [2]

- 4 A random sample of n observations ($n > 30$) from a Normally distributed population with mean μ gave the following summary statistics.

$$\sum (x - \bar{x})^2 = 78.75$$

From this sample the limits of the 95% confidence interval for μ were found to be 4.61 and 5.59

Let S^2 be the unbiased estimator of the population variance.

- (i) Show that $n = 36$ and $S^2 = 2.25$ [8]

- (ii) Find a 90% confidence interval for μ . [4]

- (iii) State, with a reason, whether your answer to part (ii) would have been different if the population was not Normally distributed. [2]

80 random samples, each of 36 observations, are taken and a 90% confidence interval for μ is calculated for each sample.

- (iv) Find the expected number of intervals that contain μ . [1]

From a random sample of 70 observations the unbiased estimator of the population variance is found to be 2.56

- (v) Using these values, find the confidence level of an interval having the same width as in part (i). [5]

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(Questions continue overleaf)

SECTION D Discrete and Decision Mathematics

Answer all six questions in this section.

- 1 (a) Consider the three graphs in Fig. 1 below.

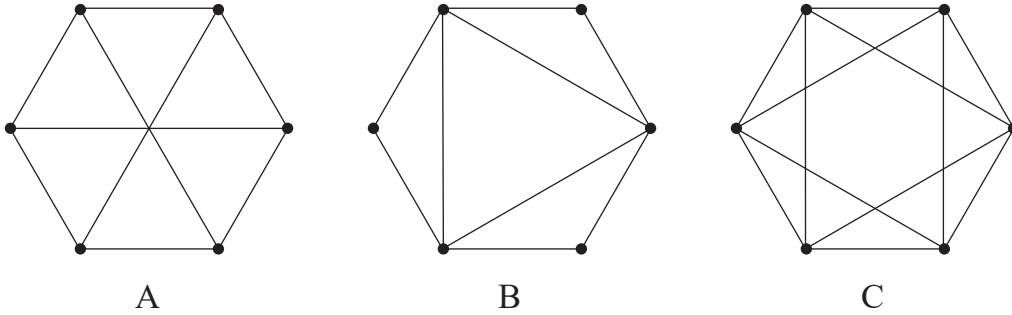


Fig. 1

Use the diagrams in the **Answer Booklet** and denote different colours by the integers 1, 2, ...

- (i) Design a minimum vertex colouring for each graph. [4]
- (ii) Design a minimum edge colouring for each graph. [4]

(b) (i) Describe a cutset of a graph G.

[2]

Fig. 2 below shows a network N, with the maximum capacities of each edge included. The flow is from vertex a to vertex h.

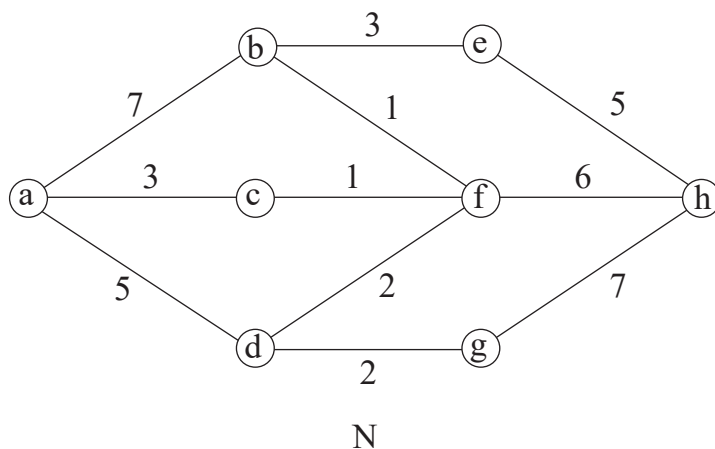


Fig. 2

(ii) Find the minimum cutset of N.

[2]

(iii) Hence find the maximum flow from vertex a to vertex h in N.

[2]

2 Fig. 3 below shows a graph G with a weight on each edge.

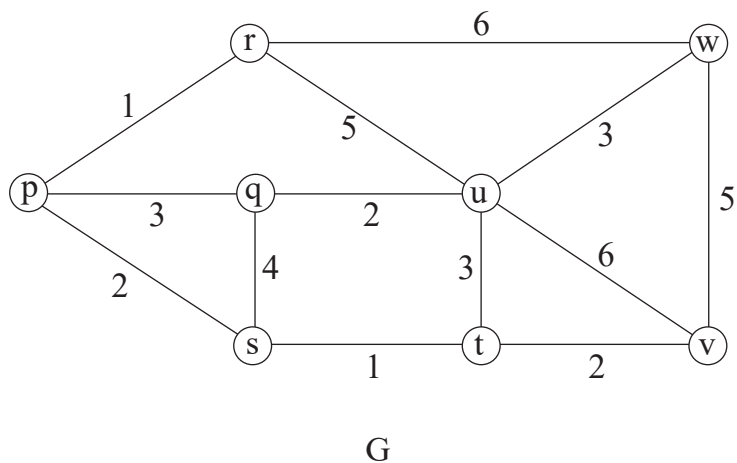


Fig. 3

(i) Apply the Nearest Neighbour Algorithm to graph G, starting at vertex s, to find a Hamiltonian path.

List the vertices in the order of output from the algorithm. [3]

(ii) Find the weight of the Hamiltonian path produced in part (i). [1]

(iii) Now apply the Nearest Neighbour Algorithm to graph G, starting at vertex v.

List the vertices in the order of output from the algorithm. [2]

(iv) State one problem with this algorithm as illustrated by the output from part (iii). [1]

3 A precedence table is shown in **Table 1** below for a construction project.

The numbers are times in weeks.

Table 1

Activity	Predecessor	Optimal time	Normal time	Pessimistic time	Expected time
A	–	11	14	17	14
B	A	7	9	11	9
C	A	8	9	13	9.5
D	A	5	7	12	7.5
E	B, C, D	10	14	15	
F	B, C, D	11	12	16	
G	E, F	$6 - d$	6	$6 + d$	6

(i) Calculate the expected times for activities E and F. [2]

(ii) Complete the PERT chart for the project in the **Answer Booklet**. [6]

(iii) List the critical path. [1]

The probability that the project is completed in less than 46 weeks is 0.95

(iv) Find the pessimistic time for activity G. [9]

- 4 The generating function for the number of ways to distribute r items between 3 boxes so that each box contains no more than 2 items is given by

$$(1 + x + x^2)^3$$

where the coefficient of x^r is the number of ways.

- (i) In how many ways can 3 items be distributed between 3 boxes, so that each box contains no more than 2 items? [5]
- (ii) Write down the generating function for the number of ways to distribute r items between 2 boxes so that each box contains between 1 and 3 items inclusive. [3]

5 Fig. 4 below shows a board.

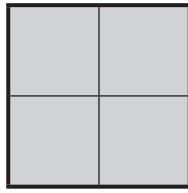


Fig. 4

(i) Find the rook polynomial for the board above.

[3]

Fig. 5 below shows a compound board.

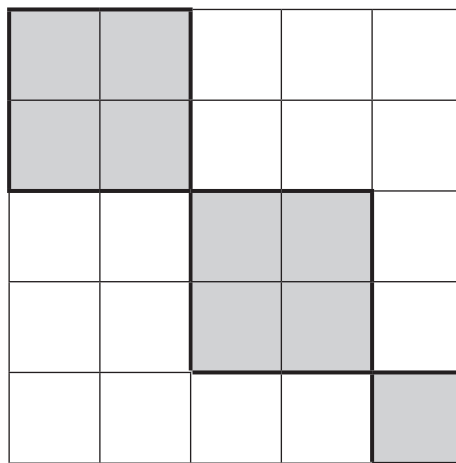


Fig. 5

(ii) Show that the rook polynomial for this compound board is

$$1 + 9x + 28x^2 + 36x^3 + 20x^4 + 4x^5$$

[6]

The local youth club is organising a fun afternoon with 5 activities, staffed by 5 helpers.

Each helper will only run one activity.

Aileen and Brian do not want to run activities 1 or 2

Claudia and Doris do not want to run activities 3 or 4

Eunice will run any activity except 5

(iii) Draw an appropriate table to show these constraints.

[3]

(iv) Using part (ii) and the Rook Inclusion Exclusion Theorem, determine the number of different ways the activities may be assigned to the helpers.

[6]

6 (a) The cycle index for the edge symmetry group of a regular tetrahedron is

$$P_G = \frac{1}{12} (x_1^6 + 8x_3^2 + 3x_1^2 x_2^2)$$

where x_q^p represents p cycles of length q .

A ceiling light is formed by joining six neon tubes in the shape of a regular tetrahedron.

If each tube is one of the three primary colours, how many different colour patterns can these six tubes have? [3]

(b) Consider the elements of the rotation group, T , of a regular tetrahedron.

Fig. 6 below shows a regular tetrahedron with its vertices labelled v_1 to v_4

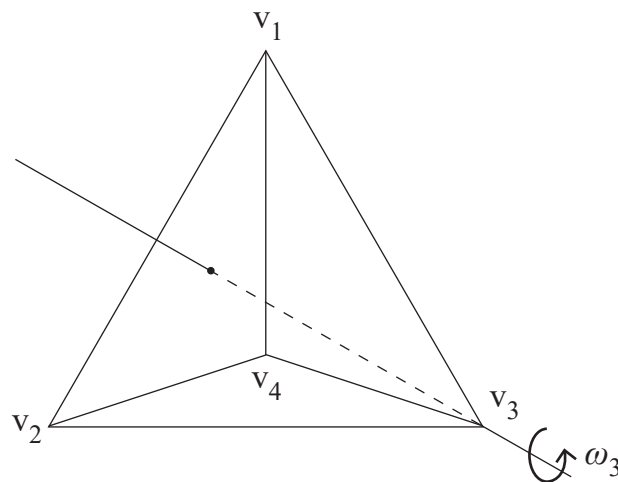


Fig. 6

ω_3 represents a rotation of the tetrahedron of 120° about the axis through v_3 and the centre of the opposite face.

(i) State the period of the element ω_3 [1]

(ii) How many elements of T , apart from the identity element, represent rotations through a vertex and the centre of its opposite face? [2]

Fig. 7 below shows an element of T , r_1 , a rotation of 180° about PQ , the centres of two opposite edges.

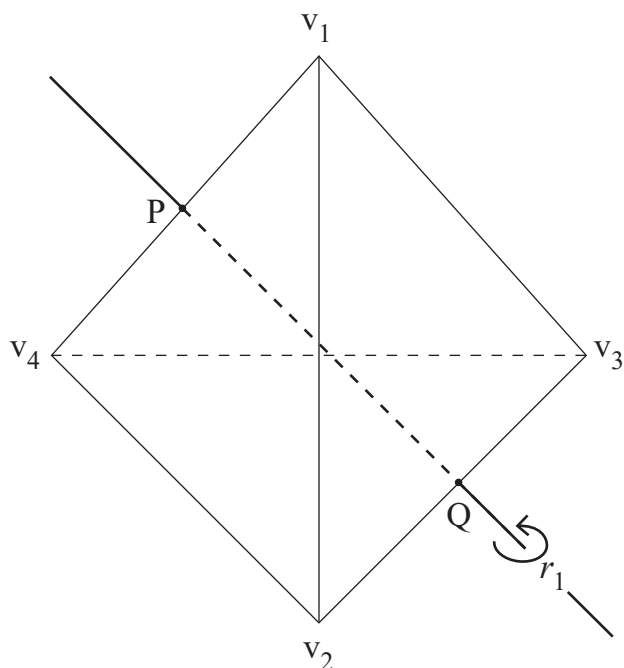


Fig. 7

Under this rotation $v_1 \leftrightarrow v_4$ $v_2 \leftrightarrow v_3$

(iii) State the period of r_1 [1]

(iv) How many elements of T , apart from the identity element, represent rotations about axes through the centres of pairs of opposite edges? [2]

(v) Find the order of T . [1]

THIS IS THE END OF THE QUESTION PAPER

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Rewarding Learning

ADVANCED
General Certificate of Education
2024

Centre Number

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Candidate Number

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ANSWER BOOKLET

Subject

Further Mathematics

Unit

2

Component Code

AFM21



AFM21

Applied Mathematics

INSTRUCTIONS TO CANDIDATES

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Question
Number

SECTION A Mechanics 1

A1

Handwriting practice area with horizontal dotted lines.





Question
Number

SECTION A Mechanics 1

A2 (i)

Area for answer A2 (i) with horizontal dotted lines for writing.

A2 (ii)

Area for answer A2 (ii) with horizontal dotted lines for writing.





Question
Number

SECTION A Mechanics 1

A2 (iii)

A2 (iv)



Question
Number

SECTION A Mechanics 1

A3 (i)

A3 (ii)





Question
Number

SECTION A Mechanics 1

A3 (iii)

A3 (iv)

Handwriting practice area with horizontal dotted lines for answers.



Question
Number

SECTION A Mechanics 1

A4 (i)

A4 (ii)

A4 (iii)



Question
Number

SECTION A Mechanics 1

A5 (i)

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Question
Number

SECTION A Mechanics 1

A5 (ii)





Question
Number

SECTION B Mechanics 2

B1 (i)



Question
Number

SECTION B Mechanics 2

B2 (i)





Question
Number

SECTION B Mechanics 2

B2 (ii)

Area for writing the answer to question B2 (ii), featuring horizontal dotted lines.



Question
Number

SECTION B Mechanics 2

B3 (i)

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Question
Number

SECTION B Mechanics 2

B3 (ii)

B3 (iii)

Handwriting practice area with horizontal dotted lines for the answer to question B3 (ii) and B3 (iii).



Question
Number

SECTION B Mechanics 2

B4 (i)

B4 (ii)





Question
Number

SECTION B Mechanics 2

B5 (i)

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Question
Number

SECTION B Mechanics 2

B5 (ii)

Area for writing the answer to question B5 (ii), featuring horizontal dotted lines.





Question
Number

SECTION C Statistics

C1 (i)





Question
Number

SECTION C Statistics

C1 (ii)

C1 (iii)

Handwriting practice area with horizontal dotted lines for the answer to question C1 (ii) and C1 (iii).





Question
Number

SECTION C Statistics

C2 (i)



Question
Number

SECTION C Statistics

C3 (i)

C3 (ii)





Question
Number

SECTION C Statistics

C4 (i)

Area for question C4 (i) with horizontal dotted lines for writing.

C4 (ii)

Area for question C4 (ii) with horizontal dotted lines for writing.





Question
Number

SECTION C Statistics

C4 (iii)

C4 (iv)

C4 (v)

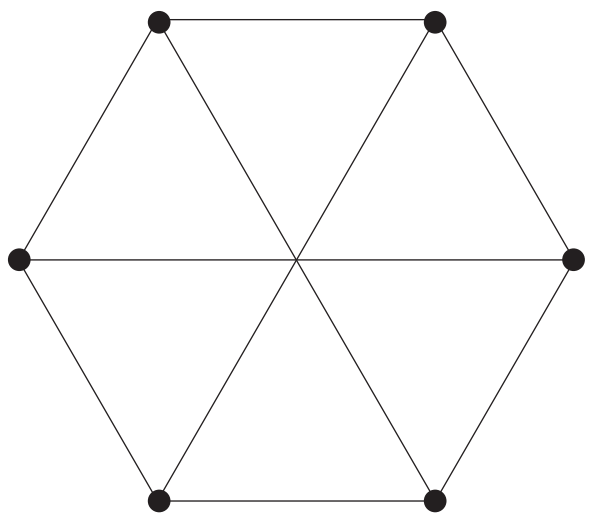
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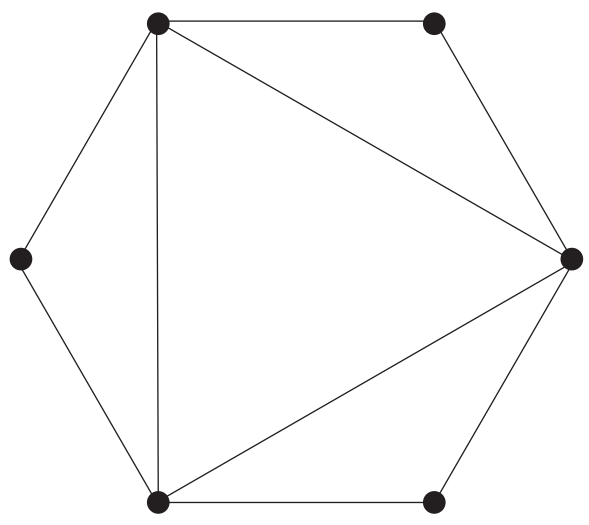
Question
Number

SECTION D Discrete and Decision Mathematics

D1 (a) (i)



A

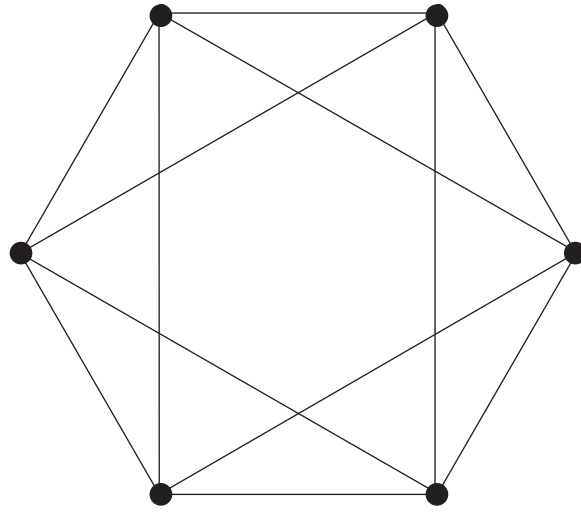


B



Question
Number

SECTION D Discrete and Decision Mathematics



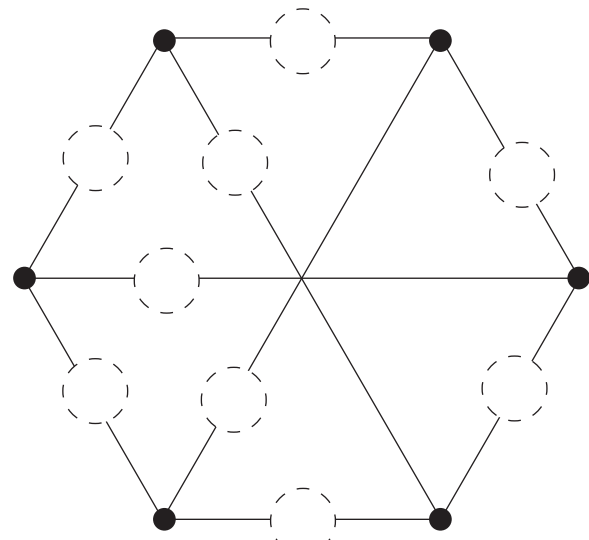
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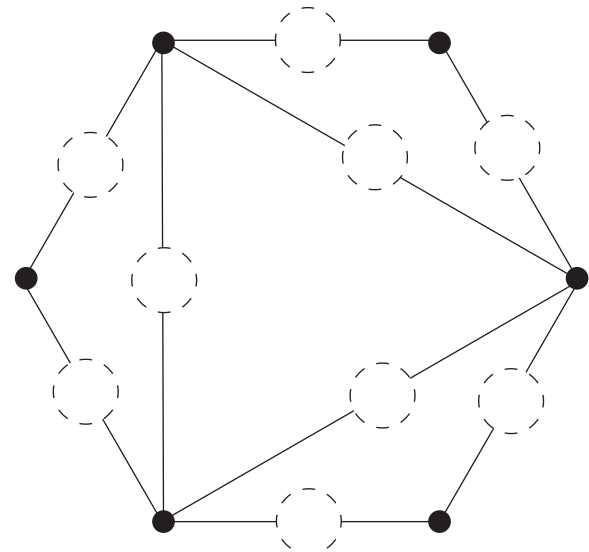
Question Number

SECTION D Discrete and Decision Mathematics

D1 (a) (ii)



A

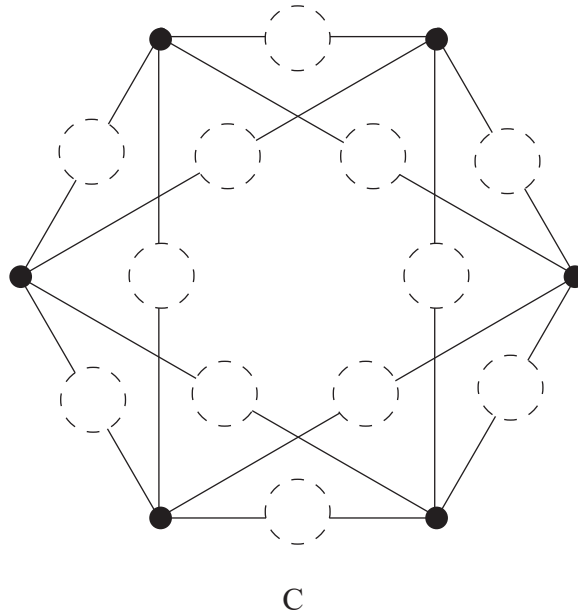


B



Question
Number

SECTION D Discrete and Decision Mathematics





Question Number

SECTION D Discrete and Decision Mathematics

D1 (b) (i)

D1 (b) (ii)

D1 (b) (iii)





Question
Number

SECTION D Discrete and Decision Mathematics

D2 (i)

D2 (ii)

D2 (iii)

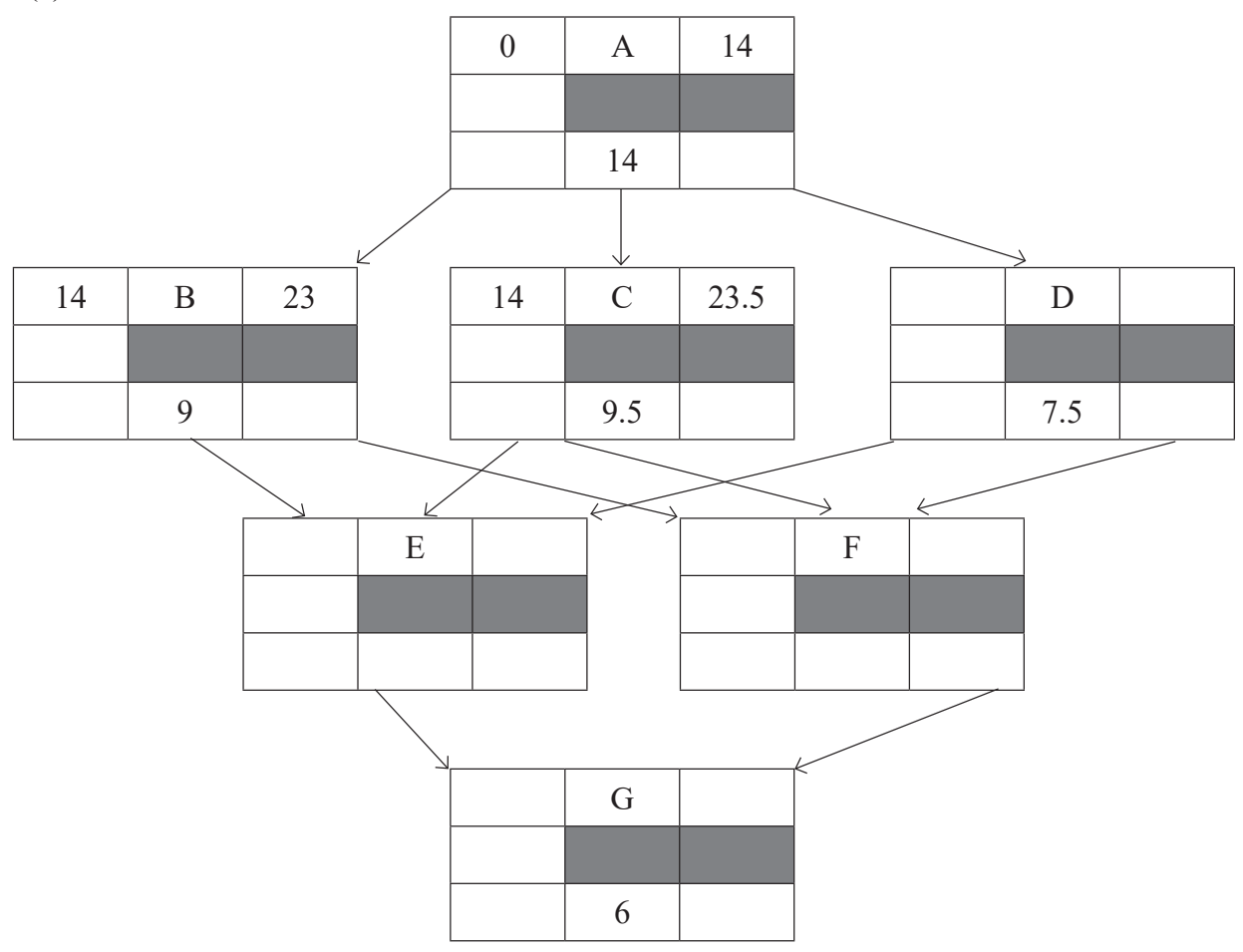
D2 (iv)

Handwriting practice area with horizontal dotted lines for each question part.



Question Number	SECTION D Discrete and Decision Mathematics
D3 (i)	

D3 (ii)



Key:

ES	Node	EF
Float		
LS	Expected time	LF

- ES = Early start time
- LS = Late start time
- EF = Early finish time
- LF = Late finish time





Question Number

SECTION D Discrete and Decision Mathematics

D4 (i)

Handwriting practice area for question D4 (i) consisting of 20 horizontal dotted lines.

D4 (ii)

Handwriting practice area for question D4 (ii) consisting of 20 horizontal dotted lines.



Question
Number

SECTION D Discrete and Decision Mathematics

D5 (i)

D5 (ii)





Question Number

SECTION D Discrete and Decision Mathematics

D5 (iii)

Handwriting practice area for question D5 (iii) with horizontal dotted lines.

D5 (iv)

Handwriting practice area for question D5 (iv) with horizontal dotted lines.



Question
Number

SECTION D Discrete and Decision Mathematics

D6 (a)





Question Number

SECTION D Discrete and Decision Mathematics

D6 (b) (i)

D6 (b) (ii)

D6 (b) (iii)

D6 (b) (iv)

D6 (b) (v)

Handwriting practice area with horizontal dotted lines for each question part.



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